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## Cross Recurrence Plot Toolbox for Matlab

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*Nonlinear Dynamics Group  
University of Potsdam*

# Reference Manual

Version 5.12, Release 25



## General Information

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**How to get:** The toolbox is freely available in the WorldWideWeb:  
<http://tocsy.agnld.uni-potsdam.de>

**How to install:** After downloading the installation script `install.m`, simply change into the folder with this file and call the command `install` from the Matlab command line. The toolbox will be automatically created and added to the `startup.m` file.

**How to deinstall:** Just call the command `crpclean` from the Matlab command line. This will remove all files of the CRP toolbox from the filesystem and its entry from the Matlab startup file.

**How to work:** This toolbox was designed for Matlab, thus one needs to install Matlab before using this toolbox.

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**Thanks:** This toolbox has been developed in the project *Nonlinear Phase and Correlation Analysis of Palaeomagnetic and Palaeoclimatic Records* under the framework of the Priority Programme *Geomagnetic variations: Spatio-temporal variations, processes and impacts on the system Earth* of the German Science Foundation (Deutsche Forschungsgemeinschaft).

**Remarks:** The toolbox is still under development. We can not give any warranty for anything related with our programmes. Please send error messages or comments to our contact address.

**Future releases:** We plan to extend the toolbox to methods of phase analysis and graphical models. Send any wishes to our contact address.

**About this document:** This manual was prepared with  $\text{\LaTeX} 2_{\epsilon}$  and using the modified `refman`-package.  
Build: December 13, 2007

## General Information

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**Warning:** Any uncritical application of the methods included in this toolbox can yield to pitfalls. The users of these programmes are urged to inform themselves by the basics of nonlinear dynamics and the problems which can occur therein.

We give no warranties for the results obtained with the toolbox.

**Theoretical Background:** The toolbox programmes are based on modern methods of nonlinear data analysis. The main focus lies on (cross) recurrence plots and their quantification. (Cross) Recurrence plots are briefly defined as

$$\mathbf{R}_{i,j}^{m,\varepsilon_i} = \Theta(\varepsilon_i - \|\vec{x}_i - \vec{x}_j\|) \quad \text{or} \quad \mathbf{CR}_{i,j}^{m,\varepsilon_i} = \Theta(\varepsilon_i - \|\vec{x}_i - \vec{y}_j\|),$$

where  $\varepsilon$  is a predefined threshold and  $\vec{x}_i, \vec{y}_i$  are phase space trajectories in an  $m$ -dimension phase space (Eckmann and Ruelle, 1987; Marwan and Kurths, 2002). These trajectories can be reconstructed from single time series  $u_i$  by using a time delay  $\tau$  (Takens, 1981)  $\hat{x}_i = (u_i, u_{i+\tau}, \dots, u_{i+(m-1)\tau})^T$ . The base of a recurrence plot is the distance matrix

$$\mathbf{D}_{i,j}^{m,\varepsilon_i} = \|\vec{x}_i - \vec{x}_j\|.$$

Another kind of recurrence plot is based on an order pattern representation of the data and is called order patterns recurrence plot (Groth, 2004). There are  $m!$  order patterns  $\pi_i$ , for example for  $m = 2$  we have

$$\pi_i = \begin{cases} 0 & \text{for } u_i < u_{i+\tau} \\ 1 & \text{for } u_i > u_{i+\tau}. \end{cases}$$

An order pattern recurrence plot is then defined as

$$\mathbf{R}_{i,j}^{m,\varepsilon_i} = \delta(\pi_i^x, \pi_j^y),$$

and should not be confused with the order matrix

$$\mathbf{O}_{i,j}^{m,\varepsilon_i} = \Theta(\|u_i - u_j\|) = \begin{cases} 0 & \text{for } u_i < u_j \\ 1 & \text{for } u_i > u_j. \end{cases}$$

The definition of the order matrix and order patterns recurrence plot can, of course, be extended to the bivariate case analogous to the cross recurrence plot.

Several quantification approaches can be applied; the most common are **recurrence rate** (Trulla et al., 1996)

$$RR = \frac{1}{N^2} \sum_{i,j=1}^N \mathbf{R}_{i,j}^{m,\varepsilon},$$

**determinism**

$$DET = \frac{\sum_{l=l_{\min}}^N l P^\varepsilon(l)}{\sum_{i,j}^N \mathbf{R}_{i,j}^{m,\varepsilon}},$$

(where  $P^\varepsilon(l) = \{l_i; i = 1 \dots N_l\}$  is the frequency distribution of the lengths  $l$  of diagonal structures and  $N_l$  is the absolute number of diagonal lines);

### **Lmax and divergence**

$$L_{max} = \max(\{l_i; i = 1 \dots N_l\}) \quad \text{respective} \quad DIV = \frac{1}{L_{max}},$$

### **entropy**

$$ENTR = - \sum_{l=l_{min}}^N p(l) \ln p(l) \quad \text{with} \quad p(l) = \frac{P^\varepsilon(l)}{\sum_{l=l_{min}}^N P^\varepsilon(l)},$$

### **laminarity (Marwan et al., 2002)**

$$LAM = \frac{\sum_{v=v_{min}}^N v P^\varepsilon(v)}{\sum_{v=1}^N v P^\varepsilon(v)},$$

(where  $P^\varepsilon(v) = \{v_i; i = 1 \dots N_v\}$  denotes the frequency distribution of the lengths  $l$  of vertical structures)

### **trapping time**

$$TT = \frac{\sum_{v=v_{min}}^N v P^\varepsilon(v)}{\sum_{v=v_{min}}^N P^\varepsilon(v)},$$

### **recurrence times of first type (Gao and Cai, 2000)**

$$T_j^1 = |\{i, j : \vec{x}_i, \vec{x}_j \in \mathcal{R}_i\}|,$$

### **recurrence times of second type**

$$T_j^2 = |\{i, j : \vec{x}_i, \vec{x}_j \in \mathcal{R}_i; \vec{x}_{j-1} \notin \mathcal{R}_i\}|$$

(where  $\mathcal{R}_i$  are the recurrence points which belong to the state  $\vec{x}_i$ ).

Above definitions are for the entire recurrence plot (or for squared windows in it, revealing some time dependencies). But most of these measures can be quantified for each diagonal line (parallel to the main diagonal) as well, which is even interesting for cross recurrence plots, for example

$$RR_k = \frac{1}{N-k} \sum_{j-i=k} \mathbf{CR}_{i,j}^{m,\varepsilon} = \frac{1}{N-k} \sum_{l=1}^{N-k} l P_k^\varepsilon(l)$$

is the recurrence rate of the  $k$ th diagonal line in the cross recurrence plot ( $P_k^\varepsilon(l) = \{l_i; i = 1 \dots N_l\}$  defines the frequency distribution of diagonal line lengths for the  $k$ th diagonal line where  $k = j - i$  in  $\mathbf{CR}_{i,j}^{m,\varepsilon}$ ).

Moreover, dynamical invariants can be estimated by using recurrence plots. At the moment, they are not yet included in this toolbox. For more details see Marwan et al. (2007).

## General Information

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The following literature is highly recommended to get introduced into nonlinear dynamics, recurrence plots and related methods and to avoid wrong usage or misinterpretation.

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**Purpose** Finds optimal transformation and maximal correlation.

**Syntax** `mcor=ace(x,y,[w,ii,oi])`  
`[theta, phi]=ace(x,y,[w,ii,oi])`  
`[theta, phi, mcor]=ace(x,y,[w,ii,oi])`  
`[theta, phi, mcor, i, o, imax, omax]=ace(x,y,[w,ii,oi])`  
`ace(...)`

**Description** Estimates the optimal transformations of the system  $\text{theta}(x)=\text{phi}(x)$  and computes the resulting maximal correlation `mcor`, where `x` is a one-column vector and `y` can be a multi-column vector.

`[theta, phi, mcor, i, o, imax, omax]=ace(x,y [w,ii,oi])` estimates the optimal transformations `theta`, `phi` and the maximal correlation `mcor` and outputs the number of inner iterations `i`, break-up number of inner inner iterations, number of outer iterations `o` and break-up number of outer inner iterations. If the algorithm doesn't converge, the number of iterations will be negative signed.

Without output arguments, `ace` plots the optimal transformations `theta` and `phi`.

**Parameters** `w` is the half-length of the boxcar window, `ii` is the maximal number of inner iterations, `oi` is the minimal number of outer iterations.

**Examples** `x=(-1:.002:1)+.3*rand(1,1001);`  
`y=(-1:.002:1).^2+.3*rand(1,1001);`  
`corrcoef(x,y)`  
`ace(y,x)`

**See Also** `mcf`

**References** Breiman, L., Friedman, J. H.: Estimating Optimal Transformations for Multiple regression and Correlation, J. Am. Stat. Assoc., 80(391), 1985.

Voss, H., Kurths, J.: Reconstruction of nonlinear time delay models from data by the use of optimal transformations, Phys. Lett. A, 234, 1997.

## adjust

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**Purpose** Adjusts two two-column vectors.

**Syntax** `[x, y]=adjust(a,b,flag)`

**Description** Adjusts the two-column vectors `a` and `b` to the same time scale (in first column), whereby using the `flag`, the following methods for adjustment can be chosen:

- 0 – (default) adjustment by cutting.
- 1 – adjustment by cubic interpolating.
- 1 – adjustment by cubic interpolating and forced length (given by `A`).
- 2 – gap filling by histogram estimation (experimental status).
- 3 – gap filling by AR(p) estimation (experimental status).

Except for `flag=0`, `x` and `y` will have the same length.

**Examples**

```
x=(1:110)';  
y1=x(11:end); y1(:,2)=sin(x(11:end)/10);  
y2=x(1:100)/2; y2(:,2)=sin(x(1:100)/5);  
[z1 z2]=adjust(y1,y2);
```

**Purpose** AR parameter estimation via Yule-Walker method.

**Syntax** `arfit(x,p)`  
`a=arfit(x,p)`  
`[a y]=arfit(x,p)`

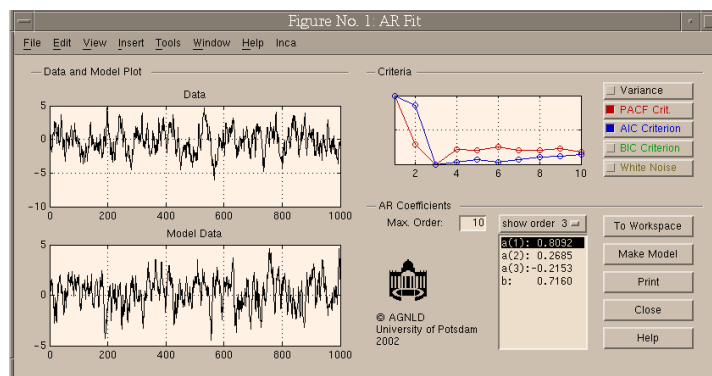
**Description** `arfit(x,p)` opens a GUI for AR coefficients estimation for the vector `x` using the Yule-Walker method. The coefficients and order selection criterias for all orders until the maximal order `p` will be solved. The coefficients are solved by the Levinson- Durbin algorithmus. The criteria are normalized in order to show them in the same plot.

`a=arfit(x,p)` returns the vector `a` of length  $(p+1)$  of the AR coefficients and the noise level for the corresponding AR model of the model order `p`. The GUI is suppressed.

`[a y]=arfit(x,p)` returns the vector `y` of a realization of the resulting AR model. The GUI is suppressed.

**Example**

```
x=rand(3,1);
a=[.8 .3 -.25 .9]';
for i=4:1000,
    x(i)=sum(a(1:3).*x(i-1:-1:i-3))+a(end)*randn;
end
arfit(x,10)
```



**Purpose** Calculate windowed cross correlation between two signals.

**Syntax** `c = corrgram(a,b,maxlag>window,noverlap)`  
`[c,l,t] = corrgram(...)`  
`c = corrgram(a,b)`  
`corrgram(a,b)`

**Description** `c = corrgram(a,b,maxlag>window,noverlap)` calculates the windowed cross correlation between the signals in vector `a` and vector `b`. `corrgram` splits the signals into overlapping segments and forms the columns of `c` with their cross correlation values up to maximum lag specified by scalar `maxlag`. Each column of `c` contains the cross correlation function between the short-term, time-localized signals `a` and `b`. Time increases linearly across the columns of `c`, from left to right. Lag increases linearly down the rows, starting at `-maxlag`. If lengths of `a` and `b` differ, the shorter signal is filled with zeros. If `n` is the length of the signals, `c` is a matrix with `2*maxlag+1` rows and

$$k = \text{fix}((n - \text{noverlap}) / (\text>window - \text{noverlap}))$$

columns.

`[c,l,t] = corrgram(...)` returns a column of lag `L` and one of time `T` at which the correlation coefficients are computed. `L` has length equal to the number of rows of `c`, `T` has length `k`.

`c = corrgram(a,b)` calculates windowed cross correlation using default settings; the defaults are `maxlag = floor(0.1n)`, `window = floor(0.1*n)` and `noverlap = 0`. You can tell `corrgram` to use the default for any parameter by leaving it off or using `[]` for that parameter, e.g. `corrgram(a,b,[],1000)`.

`corrgram(a,b)` with no output arguments plots the windowed cross correlation using the current figure.

**Example** `x = cos(0:.01:10*pi)';`  
`y = sin(0:.01:10*pi)' + .5 * randn(length(x),1);`  
`corrgram(x,y)`  
  
`corrcoeff, corr, xcorr, migram`

**Purpose** Creates a cross recurrence plot/ recurrence plot.

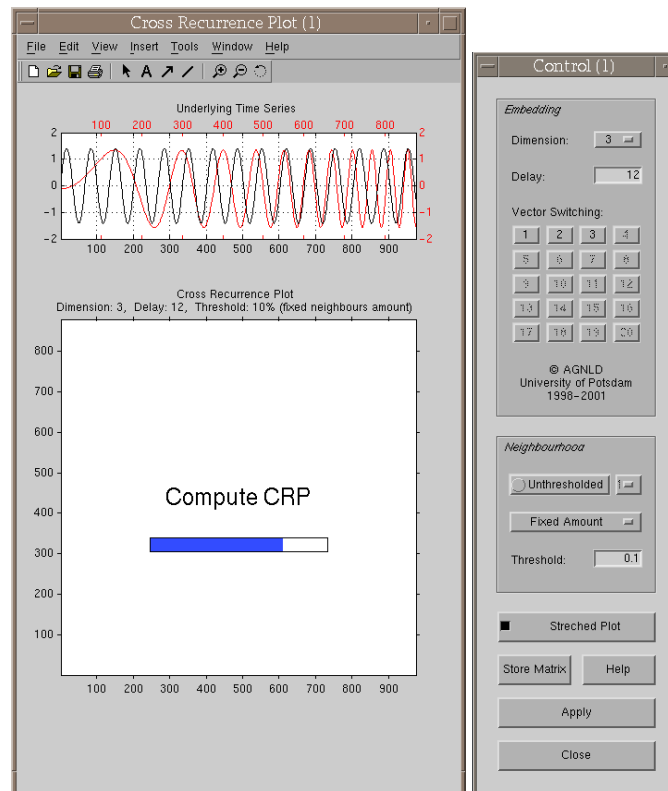
**Syntax**

```
crp(x)
crp(x,y)
crp(x,m,t,e)
r=crp(x,[],m,t,e)
r=crp(x,m,t,e,'param1','param2',...)
r=crp(x,y,m,'param1')
```

**Description** Creates a cross recurrence plot/ recurrence plot, order patterns recurrence plot as well as a distance matrix/ order matrix. Results can be stored into the workspace.

Allows to change the parameters interactively by using a GUI.

The source-data  $x$  and test-data  $y$  can be one- or a two-coloumn vectors (then, in the first column have to be the time); if the test-data  $y$  is not specified, a simple (auto) recurrence plot is created.



**Parameters** Dimension  $m$ , delay  $t$  and the size of neighbourhood  $e$  are the first three numbers after the data series; further parameters can be used to switch between various methods of finding the neighbours of the phasespace trajectory, to suppress the normalization of the data and to suppress the GUI (useful in order to use this programme by other programmes).

Methods of finding the neighbours/ of plot.

- 'maxnorm' – Maximum norm.
- 'euclidean' – Euclidean norm.
- 'minnorm' – Minimum norm.
- 'nrmnorm' – Euclidean norm between normalized vectors (all vectors have the length one).
- 'rr' – Maximum norm, fixed recurrence rate.
- 'fan' – Fixed amount of nearest neighbours.
- 'inter' – Interdependent neighbours.
- 'omatrix' – Order matrix.
- 'opattern' – Order patterns recurrence plot.
- 'distance' – Distance coded matrix (global CRP, Euclidean norm).

Normalization of the data series.

- 'normalize' – Normalization of the data.
- 'nonnormalize' – No normalization of the data.

Suppressing the GUI.

- 'gui' – Creates the GUI and the output plot.
- 'nogui' – Suppresses the GUI and the output plot.
- 'silent' – Suppresses all output.

Parameters not needed to be specified.

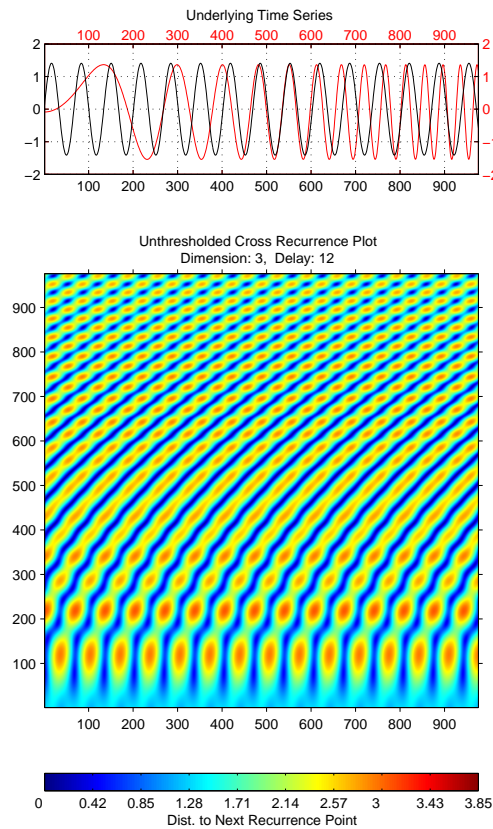
**Limitations** For higher speed in output the whole matrix of the recurrence plot is in the work space – this limits the application of long data series. However, with a little Matlab script, long data series can be handled too (cf. Examples).

**Examples**

```
a=sin((1:1000)*2*pi/67);
crp(a,'nonorm','euclidean')

X=crp(a,2,50,.1,'nogui');
spy(double(X))

b=sin(.01*([1:1000]*2*pi/67).^2);
crp(a,b,3,12,'distance')
```



For computing RPs/ CRPs of long data series, use a similar script as in the following. The data length is finally limited by the used platform performance. The examples also illustrate the capability of using the programme in a script. The first example uses sparse matrices:

```
m=3; t=20; e=.5; w=300;
x1=sin((1:5000)/40)'; x2=sin((1:7000)/80)';

clear Y, Y=spalloc(length(x2)-(m-1)*t,length(x1)-(m-1)*t,1);
k=0; h1=waitbar(0,'Compute sub CRPs - Please be patient. ');
Nx=length(x1)-(m-1)*t; Ny=length(x2)-(m-1)*t;
ax=ceil(Nx/w); ay=ceil(Ny/w);
Nx2=floor(Nx/ax); Ny2=floor(Ny/ay);
for i=1:Nx2:Nx-Nx2;
    for j=1:Ny2:Ny-Ny2, k=k+1; waitbar(k/(Nx*Ny/(Nx2*Ny2)))
        X2=crp(x1(i:i+Nx2+(m-1)*t),x2(j:j+Ny2+(m-1)*t),m,t,e,...
            'nonorm','max','silent');
        X=sparse(double(X2));
        Y(j:j+Ny2-1,i:i+Nx2-1)=X(1:Ny2,1:Nx2);
    end
end
close(h1)

spy(Y)
```

The second example writes single RPs/ CRPs to the hard disk:

```
m=3; t=20; e=.5; w=300;
x1=sin((1:5000)/40)'; x2=sin((1:7000)/80)';
Nx=length(x1); Ny=length(x2);

% compute single CRPs and write them to the hard disk
b1=zeros((m-1)*t+ceil(length(x1)/w)*w,1);
b1(1:length(x1))=x1;
b2=zeros((m-1)*t+ceil(length(x2)/w)*w,1);
b2(1:length(x2))=x2;
h=waitbar(0,'Compute sub CRPs - Please be patient.')
for i=1:w:length(b1)-w-1, waitbar(i/((length(b1)-w-1)))
    for j=1:w:length(b2)-w-1,j
        X=crp(b1(i:i+w+(m-1)*t-1),b2(j:j+w+(m-1)*t-1),m,t,e,...
            'max','silent','nonorm');
        i2=num2str((i+w-1)/w);j2=num2str((j+w-1)/w);
        filename=['CRP_',i2,'_',j2,'.tif'];
        imwrite(X,filename,'tif')
    end
end, close(h)

% read single CRPs and unify them
xmax=(i+w-1)/w; ymax=(j+w-1)/w;
clear Y, h=waitbar(0,'Read sub CRPs - Please be patient.');
for i=1:xmax,waitbar(i/xmax)
    for j=1:ymax,
        i2=num2str(i);j2=num2str(j);
        filename=['CRP_',i2,'_',j2,'.tif'];
        X=imread(filename,'tif');
        Y(i*w-(w-1):i*w,j*w-(w-1):j*w)=(X)';
    end
end, close(h)

Y(Nx+1:end,:)=[]; Y(:,Ny+1:end)=[];
spy(double(Y))
```

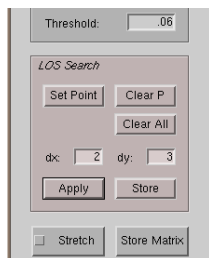
**See Also** crp2, crp\_big, crqa

**Purpose** Creates a cross recurrence plot/ recurrence plot and computes the line of synchronization.

**Syntax**

```
crp2(x)
crp2(x,y)
crp2(x,m,t,e)
r=crp2(x,[],m,t,e)
r=crp2(x,m,t,e,'param1','param2',...)
r=crp2(x,y,m,'param1')
```

**Description** Creates a cross recurrence plot/ recurrence plot, order patterns recurrence plot as well as a distance matrix from the embedding vectors  $x$  and  $y$ . Results can be stored into the workspace. Further it is possible to estimate the line of synchronization (LOS) in order to get the nonparametric time-relationship between the two considered systems.



Allows to change the parameters interactively by using a GUI.

The embedding dimension  $m$  is given by the size of the  $n \times m$  matrix  $x$  and  $y$ ; if the matrix  $y$  is not specified, a simple (auto) recurrence plot is created.

**Parameters** Additionally dimension  $m$ , delay  $t$  and the size of neighbourhood  $e$  are the first three numbers after the data series; further parameters can be used to switch between various methods of finding the neighbours of the phasespace trajectory, to suppress the normalization of the data and to suppress the GUI (useful in order to use this programme by other programmes).

Methods of finding the neighbours.

- 'maxnorm' – Maximum norm.
- 'euclidean' – Euclidean norm.
- 'minnorm' – Minimum norm.
- 'nrmnorm' – Euclidean norm between normalized vectors (all vectors have the length one).
- 'rr' – Maximum norm, fixed recurrence rate.
- 'fan' – Fixed amount of nearest neighbours.
- 'omatrix' – Order matrix (disabled).
- 'opattern' – Order patterns recurrence plot.
- 'distance' – Distance coded matrix (global CRP, Euclidean norm).

Normalization of the data series.

'normalize' – Normalization of the data.

'nonnormalize' – No normalization of the data.

Suppressing the GUI.

'gui' – Creates the GUI and the output plot.

'nogui' – Suppresses the GUI and the output plot.

'silent' – Suppresses all output.

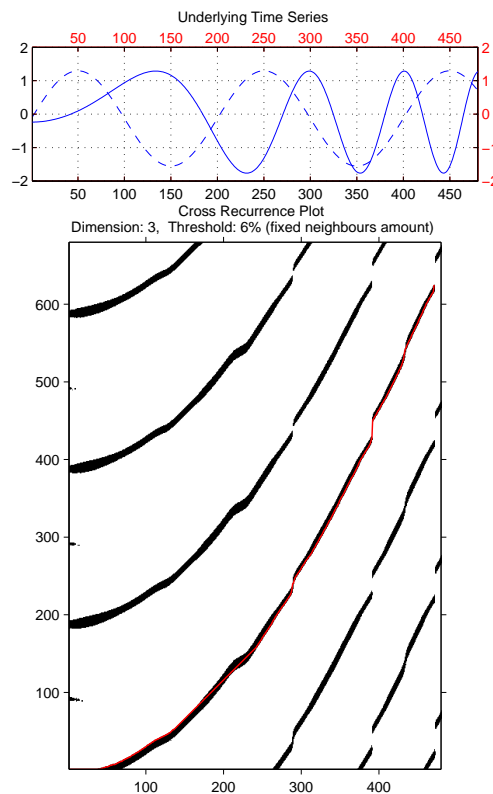
Parameters not needed to be specified.

**Limitations** For higher speed in output the whole matrix of the recurrence plot is in the work space – this limits the application of long data series. However, a solution for using long data you can find under the description for `crp`.

**Examples**

```
a=sin((1:1000)*2*pi/200);    % pendulum's location vector
b=cos((1:1000)*2*pi/200);    % pendulum's velocity vector
crp2(a(1:500),b(1:500),'nonorm','euclidean')
```

```
b=sin(.01*([1:1000]*2*pi/67).^2);
crp2(b(1:500),a(1:700),3,10,.06,'fan')
```



**See Also** `crp`, `crp_big` and `trackplot`

**References** Marwan, N., Thiel, M., Nowaczyk, N.: Cross Recurrence Plot Based Synchronization of Time Series, *Nonlin. Proc. Geophys.*, 9, 2002.

**Purpose** Creates a cross recurrence plot/ recurrence plot.

**Syntax**

```
crp_big(x)
crp_big(x,y)
crp_big(x,m,t,e)
r=crp(x,[],m,t,e)
r=crp(x,m,t,e,'param1','param2',...)
r=crp(x,y,m,'param1')
```

**Description** Creates a cross recurrence plot/ recurrence plot, order patterns recurrence plot as well as a distance matrix/ order matrix. In contrast to CRP, long data series can be used. Results can be stored into the workspace.

Allows to change the parameters interactively by using a GUI.

The source-data *x* and test-data *y* can be one- or a two-coloumn vectors (then, in the first column have to be the time); if the test-data *y* is not specified, a simple (auto) recurrence plot is created.

**Parameters** Dimension *m*, delay *t* and the size of neighbourhood *e* are the first three numbers after the data series; further parameters can be used to switch between various methods of finding the neighbours of the phasespace trajectory, to suppress the normalization of the data and to suppress the GUI (useful in order to use this programme by other programmes).

Methods of finding the neighbours/ of plot.

'maxnorm'	– Maximum norm.
'euclidean'	– Euclidean norm.
'minnorm'	– Minimum norm.
'nrmnorm'	– Euclidean norm between normalized vectors (all vectors have the length one).
'rr'	– Maximum norm, fixed recurrence rate.
'fan'	– Fixed amount of nearest neighbours.
'inter'	– Interdependent neighbours.
'omatrix'	– Order matrix.
'opattern'	– Order patterns recurrence plot.
'distance'	– Distance coded matrix (global CRP, Euclidean norm).

Normalization of the data series.

'normalize'	– Normalization of the data.
'nonnormalize'	– No normalization of the data.

Suppressing the GUI.

'gui'	– Creates the GUI and the output plot.
'nogui'	– Suppresses the GUI and the output plot.
'silent'	– Suppresses all output.

Parameters not needed to be specified.

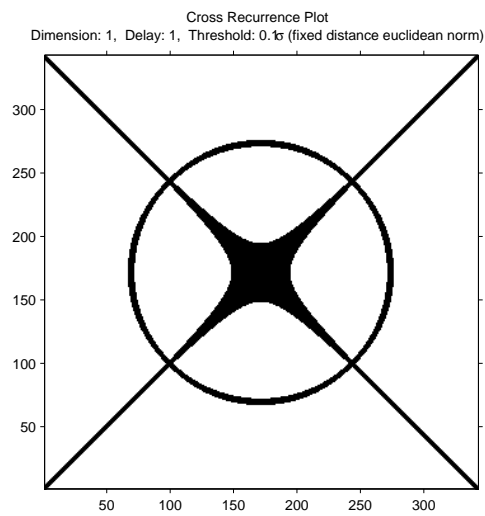
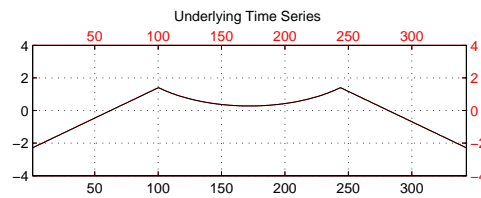
**Limitations** In contrast to *crp* and *crp2*, this command allows to work with longer data series. The algorithm computes the CRP piecewise. However, the possibility to store the CRP in the workspace limits the length of data series again. However, a solution for using long data you can find under the description for *crp*.

**Examples**

```

a=sqrt(100^2-(-71:71).^2); b=1:100;
b(101:100+length(a))=-(a)+170;
b(end+1:end+100)=100:-1:1;
crp_big(b,1,1,.1,'euclidean')

```



**See Also** `crp`, `crp2` and `crqa`

**Purpose** Computes and plots the CRQA measures.

**Syntax**

```
crqa(x)
crqa(x,y)
y=crqa(x,y,m,t,e,w,ws)
y=crqa(x,y,m,t,e,w,ws,lmin,vmin)
y=crqa(x,y,m,t,e,w,ws,lmin,vmin,tw)
y=crqa(x,y,m,t,e,[],'param1','param2',...)
```

**Description** Recurrence quantification analysis of cross-recurrence with the first vector  $x$  and the second  $y$ .

The input vectors can be multi-column vectors, where each column will be used as a component of the phase-space vector. However, if the first column is monotonically increasing, it will be used as an time scale for plotting.

**Parameters** CRQA(...) without any output arguments opens a GUI for interactively control the CRQA. If an output is specified with using the option 'gui', then the output will contain the figure handle.

Dimension  $m$ , delay  $t$ , the size of neighbourhood  $e$ , the window size  $w$  and the shift value  $ws$  are the first five numbers after the data series; if  $w=[]$  then the whole plot will be calculated. The minimal length of diagonal and vertical structures can be specified with  $lmin$  and  $vmin$  respectively (default is 2).

As the last numeric parameter, the size of the Theiler window  $tw$  can be specified (default is 1). This window excludes the recurrence points parallel to the main diagonal from the analysis. The application of the Theiler window is useful only for recurrence plots. In cross recurrence plots, the size of the Theiler window will be set automatically to zero.

Further parameters can be used to switch between various methods of finding the neighbours of the phasespace trajectory, to suppress the normalization of the data and to suppress the GUI (useful in order to use this programme by other programmes).

Methods of finding the neighbours.

'maxnorm'	– Maximum norm.
'euclidean'	– Euclidean norm.
'minnorm'	– Minimum norm.
'nrmnorm'	– Euclidean norm between normalized vectors (all vectors have the length one).
'rr'	– Maximum norm, fixed recurrence rate.
'fan'	– Fixed amount of nearest neighbours.
'inter'	– Interdependent neighbours.
'omatrix'	– Order matrix.
'opattern'	– Order patterns recurrence plot.

Normalization of the data series.

'normalize'	– Normalization of the data.
'nonnormalize'	– No normalization of the data.

#### Suppressing the GUI.

- 'gui' – Creates the GUI.
- 'nogui' – Suppresses the GUI.
- 'silent' – Suppresses all output.

#### Output

- y(:,1) – Recurrence rate.
- y(:,2) – Determinism.
- y(:,3) – Averaged diagonal length.
- y(:,4) – Length of longest diagonal line.
- y(:,5) – Entropy of diagonal length.
- y(:,6) – Laminarity.
- y(:,7) – Trapping time.
- y(:,8) – Length of longest vertical line.
- y(:,9) – Recurrence time of 1st type.
- y(:,10) – Recurrence time of 2nd type.

Parameters not needed to be specified.

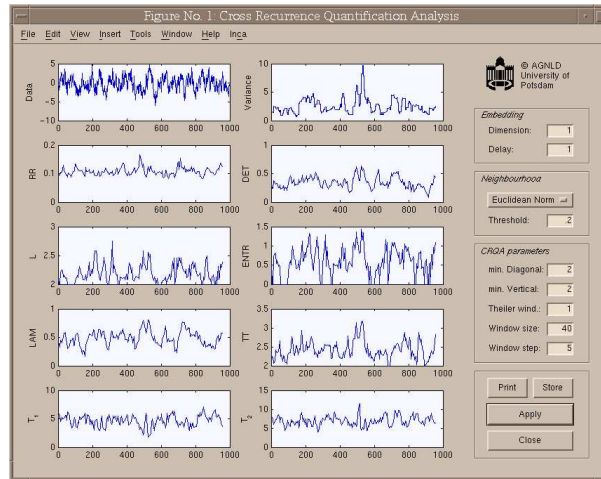
The window of length  $w$  is applied on the data and not on the RP, i. e. the RP will have smaller size than the window, thus  $w - (m - 1)\tau$ . If we consider the data window at time  $i \dots i + w$ , the corresponding RQA measures are assigned to time  $i$ . Therefore, if you see a beginning of a transition in the plot of the RQA measures at time  $i$ , this transition will probably happen at time  $i + w - (m - 1)\tau$ .

**Limitations** For higher speed in output the whole matrix of the recurrence plot is in the work space – this limits the application of long data series. However, a solution for using long data you can find under the description for `crp`.

**Warning** The RQA measures may differ from those of the RQA programmes by Charles Webber Jr. For compatibility use a Theiler window of size one and ensure that the data are normalized before by the same distance which is used in the RQA programmes; e. g. normalize with the maximal phase space diameter, which can be estimated with the programme `pss`:

```
RQA=crqa(100*x/pss(x,dim,lag,'euclidean'),...
        dim,lag,e,[],[],l_min,v_min,1,...
        'euclidean','nonnormalize','silent')
```

**Examples** `a=randn(300,1);`  
`crqa(a,1,1,.2,40,2,'euc')`



```
N=300; w=40; ws=2;
a=3.4:.6/(N-1):4;
b=.5; for i=2:N, b(i)=a(i)*b(i-1)*(1-b(i-1));end
y=crqa(b,3,2,.1,w,ws);
subplot(2,1,1), plot(a,b,'.','markersize',.1)
title('logistic map'), axis([3.4 4 0 1])
subplot(2,1,2), plot(a(1:ws:N-w),y(1:ws:N-w,1))
ylabel('recurrence rate'), axis([3.4 4 0 1])
```

**See Also** `crqad`, `crp`, `dl`, `tt`, `pss`

**References** Trulla, L. L., Giuliani, A., Zbilut, J. P., Webber Jr., C. L.: Recurrence quantification analysis of the logistic equation with transients, *Phys. Lett. A*, 223, 1996.

Marwan, N., Wessel, N., Meyerfeldt, U., Schirdewan, A., Kurths, J.: Recurrence Plot Based Measures of Complexity and its Application to Heart Rate Variability Data, *Phys. Rev. E*, 66(2), 2002.

Gao, J. B.: Recurrence Time Statistics for Chaotic Systems and Their Applications, *Phys. Rev. Lett.*, 83(16), 1999.

Thiel, M., Romano, M. C., Kurths, J., Meucci, R., Allaria, E., Arecchi, F. T.: Influence of observational noise on the recurrence quantification analysis, *Physica D*, 17(3), 2002.

**Purpose** Computes and plots the diagonalwise CRQA measures.

**Syntax** `crqad(x)`  
`crqad(x,y)`  
`y=crqad(x,y,m,t,e,w)`  
`y=crqad(x,y,m,t,e,[],'param1','param2',...)`

**Description** Recurrence quantification analysis of diagonals in the cross recurrence plot of the vectors  $x$  and  $y$  as well as  $x$  and  $-y$  for the diagonals within the range  $[-w,w]$  around the main diagonal. The output is a structure (see below).

**Parameters** Dimension  $m$ , delay  $t$ , the size of neighbourhood  $e$  and the window size  $w$  are the first five numbers after the data series; if  $w=[]$  then the whole plot will be calculated. Further parameters can be used to switch between various methods of finding the neighbours of the phasespace trajectory, to suppress the normalization of the data and to suppress the GUI (useful in order to use this programme by other programmes). The minimal length of diagonal and vertical structures can be setted only in the GUI.

Methods of finding the neighbours.

'maxnorm'	– Maximum norm.
'euclidean'	– Euclidean norm.
'minnorm'	– Minimum norm.
'nrmnorm'	– Euclidean norm between normalized vectors (all vectors have the length one).
'rr'	– Maximum norm, fixed recurrence rate.
'fan'	– Fixed amount of nearest neighbours.
'inter'	– Interdependent neighbours.
'omatrix'	– Order matrix.
'opattern'	– Order patterns recurrence plot.

Normalization of the data series.

'normalize'	– Normalization of the data.
'nonnormalize'	– No normalization of the data.

Suppressing the GUI.

'gui'	– Creates the GUI.
'nogui'	– Suppresses the GUI.
'silent'	– Suppresses all output.

Output

<code>y.RRp</code>	– Recurrence rate $(x,y)$ .
<code>y.RRp</code>	– Recurrence rate $(x,-y)$ .
<code>y.DETp</code>	– Determinism $(x,y)$ .
<code>y.DETm</code>	– Determinism $(x,-y)$ .
<code>y.Lp</code>	– Averaged diagonal length $(x,y)$ .
<code>y.Lm</code>	– Averaged diagonal length $(x,-y)$ .

Parameters not needed to be specified.

**Limitations** For higher speed in output the whole matrix of the recurrence plot is in the work space – this limits the application of long data series. However, a solution for using long data you can find under the description for `crp`.

**Examples** `a=sin(0:.1:80)+randn(1,801);`  
`b=sin(0:.1:80)+randn(1,801);`  
`crqad(a,b,3,15,.1,100,'fan')`

**See Also** `crqad_big`, `crqa`, `crp`, `dl`, `tt`

**References** Marwan, N., Kurths, J.: Nonlinear analysis of bivariate data with cross recurrence plots, Phys. Lett. A, 302, 2002.

**Purpose** Computes and plots the diagonalwise CRQA measures of long data series.

**Syntax** `crqad_big(x)`  
`crqad_big(x,y)`  
`y=crqad_big(x,y,m,t,e,w)`  
`y=crqad_big(x,y,m,t,e,[],'param1','param2',...)`

**Description** Recurrence quantification analysis of diagonals in the cross recurrence plot of the vectors  $x$  and  $y$  as well as  $x$  and  $-y$  for the diagonals within the range  $[-w,w]$  around the main diagonal. The output is a structure (see below).

**Parameters** Dimension  $m$ , delay  $t$ , the size of neighbourhood  $e$  and the window size  $w$  are the first five numbers after the data series; if  $w=[]$  then the whole plot will be calculated. Further parameters can be used to switch between various methods of finding the neighbours of the phasespace trajectory, to suppress the normalization of the data and to suppress the GUI (useful in order to use this programme by other programmes). The minimal length of diagonal and vertical structures can be setted only in the GUI.

Methods of finding the neighbours.

'maxnorm' – Maximum norm.  
'euclidean' – Euclidean norm.  
'minnorm' – Minimum norm.

Normalization of the data series.

'normalize' – Normalization of the data.  
'nonnormalize' – No normalization of the data.

Suppressing the GUI.

'gui' – Creates the GUI.  
'nogui' – Suppresses the GUI.  
'silent' – Suppresses all output.

Output

$y.RRp$  – Recurrence rate  $(x,y)$ .  
 $y.RRp$  – Recurrence rate  $(x,-y)$ .  
 $y.DETp$  – Determinism  $(x,y)$ .  
 $y.DETm$  – Determinism  $(x,-y)$ .  
 $y.Lp$  – Averaged diagonal length  $(x,y)$ .  
 $y.Lm$  – Averaged diagonal length  $(x,-y)$ .

Parameters not needed to be specified.

## crqad\_big

---

**Limitations** In contrast to `crqad`, only maximum, Euclidean and minimum norm are available.

**Examples**

```
a=sin(0:.1:800)+randn(1,8001);  
b=sin(0:.1:800)+randn(1,8001);  
crqad_big(a,b,3,15,.1,50,'euc')
```

**See Also** `crqa`, `crqad`, `crp`, `dl`, `tt`

**References** Marwan, N., Kurths, J.: Nonlinear analysis of bivariate data with cross recurrence plots, Phys. Lett. A, 302, 2002.

**Purpose** Mean of the diagonal line lengths and their distribution.

**Syntax** `a=dl(x)`  
`[a b]=dl(x)`

**Description** `a=dl(x)` computes the mean of the length of the diagonal line structures in a recurrence plot.

`[a b]=dl(x)` computes the mean `a` and the lengths of the of the found diagonal lines, stored in `b`. In order to get the histogramme of the line lengths, simply call `hist(b,[1 max(b)])`.

**See Also** `crqa`, `crqaplot`, `tt`

## entropy

---

**Purpose** Entropy of a distribution.

**Syntax** `e=entropy(h)`

**Description** Computes the entropy of the distribution `h`.

**Examples** `x=randn(100,1);`  
`h=hist(x);`  
`entropy(h')`

- Purpose** Find the optimal embedding dimension by means of false nearest neighbours.
- Syntax** `y=fnn(x)`  
`y=fnn(x,m)`  
`y=fnn(x,m,t)`  
`y=fnn(x,m,t,r,s)`  
`fnn(...)`  
`fnn(...,param)`
- Description** Computes the amount  $y$  of false nearest neighbours (FNN) as a function of the embedding dimension. The optimal embedding is then chosen as the one where the amount of FNNs almost vanishes.
- `fnn(...)` without any output arguments opens a GUI for interactively changing the parameters.
- By using the GUI, the FNN can be stored into the workspace.
- `fnn` without any arguments calls a demo (the same as the example below).
- Parameters** The parameters maximal dimension  $m$  (default 10), delay  $t$  (default 1), neighbourhood criterion  $r$  (default 2), size of the neighbourhood  $s$  (default Inf) and maximal number of random samples  $n$  (default `length(x)` if the data length is smaller than 500, else 200) are optional.
- Additional parameters according to the GUI.
- 'gui' – Creates the GUI.
  - 'nogui' – Suppresses the GUI.
  - 'silent' – Suppresses all output.
- Parameters not needed to be specified.
- Examples** `x=sin(0:.2:8*pi)'+.1*randn(126,1);`  
`fnn(x,10,[],5)`
- See Also** `phasespace`, `pss`, `mi`
- References** Kennel, M. B., Brown, R., Abarbanel, H. D. I.: Determining embedding dimension for phase-space reconstruction using a geometrical construction, Phys. Rev. A, 45, 1992.

## hist2

---

**Purpose** Creates a two dimensional histogram.

**Syntax** `p=hist2(x)`  
`p=hist2(x,y)`  
`p=hist2(x,k,l)`  
`[p,j]=hist2(...)`  
`hist2(...)`  
`hist2(...,'gui')`

**Description** `p=hist2(x)` bins the two-dimensional density of  $x(i)$  and  $x(i+1)$  into a 10x10 equally spaced matrix and returns it in `p`.

`p=hist2(x,y)` bins the two-dimensional density of  $x_i$  and  $y_i$  into a 10x10 equally spaced matrix and returns it in `p`.

`p=hist2(x,k,l)`, where `k` and `l` are scalars, uses `k` bins and a lag `l`.

`[p,j]=hist2(...)` returns the matrix `p` and the two-column vector `j` containing the two-dimensional density matrix and the bin location for `x` (and `y`).

`hist2(...)` without any output arguments produces a histogram plot.

`hist2(...,'gui')` creates a GUI for interactively changing of the parameters.

**Examples** `x=randn(10000,1);`  
`hist2(x)`

**See Also** `histn`, `mi`

**Purpose** Creates a multi-dimensional histogram.

**Syntax** `p=histn(x)`  
`p=histn(x1,...,xN)`  
`p=histn([x,...,xN])`  
`p=histn(x,l)`  
`[p,j]=histn(...)`  
`histn(...)`

**Description** `p=histn(x)` bins the two-dimensional density of  $x(i)$  and  $x(i+1)$  into a 10x10 equally spaced matrix and returns it in `p` (this is similar to `hist2`).  
`p=histn(x1,x2,...,xN)` or `p=hist2([x1,x2,...,xN])` bins the N-dimensional density of  $x_i$  into a 10x10 equally spaced matrix and returns it in `p`. Since both variants of input the arguments can be combined, the usage of various multi-column vectors is possible; the dimension is the sum of the number of all vectors' columns.  
`p=histn(x,l)`, where `l` is a scalar, uses a lag `l`.  
`p=histn(x,k,l)`, where `k` and `l` are scalars, uses `k` bins and a lag `l`.  
`[p,j]=histn(...)` returns the N-dimensional matrix `p` and the two-column vector `j` containing the N-dimensional density matrix and the bin location for `x1,...,xN`.  
`histn(...)` without any output arguments produces a histogram plot.

**Examples** `x=randn(10000,3);`  
`histn(x)`

**See Also** `hist2`, `mi`

**Purpose** Creates a joint recurrence plot.

**Syntax** `jrp(x)`  
`jrp(x,y)`  
`jrp(x,m,t,e)`  
`r=jrp(x,[],m,t,e)`  
`r=jrp(x,m,t,e,'param1','param2',...)`  
`r=jrp(x,y,m,'param1')`

**Description** Creates a simple joint recurrence plot of maximal two data series, based on different norms or recurrence plots. Embedding parameters will be the same for both systems. Results can be stored into the workspace.

Allows to change the parameters interactively by using a GUI.

The source-data *x* and test-data *y* can be one- or a two-coloumn vectors (then, in the first column have to be the time); if the test-data *y* is not specified, a simple (auto) recurrence plot is created.

**Parameters** Dimension *m*, delay *t* and the size of neighbourhood *e* are the first three numbers after the data series; further parameters can be used to switch between various methods of finding the neighbours of the phasespace trajectory, to suppress the normalization of the data and to suppress the GUI (useful in order to use this programme by other programmes).

Methods of finding the neighbours/ of plot.

'maxnorm'	– Maximum norm.
'euclidean'	– Euclidean norm.
'minnorm'	– Minimum norm.
'nrmnorm'	– Euclidean norm between normalized vectors (all vectors have the length one).
'rr'	– Maximum norm, fixed recurrence rate.
'fan'	– Fixed amount of nearest neighbours.
'inter'	– Interdependent neighbours.
'omatrix'	– Order matrix.
'opattern'	– Order patterns recurrence plot.
'distance'	– Distance coded matrix (global JRP, Euclidean norm).

Normalization of the data series.

'normalize'	– Normalization of the data.
'nonnormalize'	– No normalization of the data.

Suppressing the GUI.

'gui'	– Creates the GUI and the output plot.
'nogui'	– Suppresses the GUI and the output plot.
'silent'	– Suppresses all output.

Parameters not needed to be specified.

**Limitations** For higher speed in output the whole matrix of the recurrence plot is in the work space – this limits the application of long data series. However, with a little Matlab script, long data series can be handled too (cf. Examples for `crp`).

**Examples** `a=sin((1:1000)*2*pi/67);`  
`b=sin(.01*([1:1000]*2*pi/67).^2);`  
`jrp(a,b,3,12,'fan')`

**See Also** `crp`, `jrqa`

**References** Romano, M., Thiel, M., Kurths, J., von Bloh, W.: Multivariate Recurrence Plots, *Phys. Lett. A* , 330, 2004.

**Purpose** Computes and plots the JRQA measures.

**Syntax** `jrqa(x)`  
`jrqa(x,y)`  
`y=jrqa(x,y,m,t,e,w,ws)`  
`y=jrqa(x,y,m,t,e,w,ws,lmin,vmin)`  
`y=jrqa(x,y,m,t,e,w,ws,lmin,vmin,tw)`  
`y=jrqa(x,y,m,t,e,[],'param1','param2',...)`

**Description** Recurrence quantification analysis of joint-recurrence plots with the first vector *x* and the second *y*.

The input vectors can be multi-column vectors, where each column will be used as a component of the phase-space vector. However, if the first column is monotonically increasing, it will be used as an time scale for plotting.

**Parameters** `JRQA(...)` without any output arguments opens a GUI for interactively control the JRQA. If an output is specified with using the option `'gui'`, then the output will contain the figure handle.

Dimension *m*, delay *t*, the size of neighbourhood *e*, the window size *w* and the shift value *ws* are the first five numbers after the data series; if *w*=[] then the whole plot will be calculated. The minimal length of diagonal and vertical structures can be specified with *lmin* and *vmin* respectively (default is 2).

As the last numeric parameter, the size of the Theiler window *tw* can be specified (default is 1). This window excludes the recurrence points parallel to the main diagonal from the analysis. The application of the Theiler window is useful only for recurrence plots. In joint recurrence plots, the size of the Theiler window will be set automatically to zero.

Further parameters can be used to switch between various methods of finding the neighbours of the phasespace trajectory, to suppress the normalization of the data and to suppress the GUI (useful in order to use this programme by other programmes).

Methods of finding the neighbours.

- `'maxnorm'` – Maximum norm.
- `'euclidean'` – Euclidean norm.
- `'minnorm'` – Minimum norm.
- `'nrmnorm'` – Euclidean norm between normalized vectors (all vectors have the length one).
- `'rr'` – Maximum norm, fixed recurrence rate.
- `'fan'` – Fixed amount of nearest neighbours.
- `'inter'` – Interdependent neighbours.
- `'omatrix'` – Order matrix.
- `'opattern'` – Order patterns recurrence plot.

Normalization of the data series.

- `'normalize'` – Normalization of the data.
- `'nonnormalize'` – No normalization of the data.

**Suppressing the GUI.**

'gui'            – Creates the GUI.  
'nogui'         – Suppresses the GUI.  
'silent'        – Suppresses all output.

**Output**

y(:,1)           – Recurrence rate.  
y(:,2)           – Determinism.  
y(:,3)           – Averaged diagonal length.  
y(:,4)           – Length of longest diagonal line.  
y(:,5)           – Entropy of diagonal length.  
y(:,6)           – Laminarity.  
y(:,7)           – Trapping time.  
y(:,8)           – Length of longest vertical line.  
y(:,9)           – Recurrence time of 1st type.  
y(:,10)          – Recurrence time of 2nd type.

Parameters not needed to be specified.

**Limitations** For higher speed in output the whole matrix of the recurrence plot is in the work space – this limits the application of long data series. However, a solution for using long data you can find under the description for `crp`.

**Examples** `N=500; w=40; ws=10;  
b=.4; a=.6; mu=.8:-0.7/N:.1;  
  
% two mutually coupled logistic maps  
for i=2:N,  
    a(i)=3.6*a(i-1)*(1-a(i-1));  
    b(i)=4*b(i-1)*(1-b(i-1))-mu(i)*a(i);  
end  
  
% coupling is obtained by higher RR and DET values  
jrqa(a,b,1,1,.2,w,ws);`

**See Also** `crqa`, `jrp`, `crp`

**References** Trulla, L. L., Giuliani, A., Zbilut, J. P., Webber Jr., C. L.: Recurrence quantification analysis of the logistic equation with transients, *Phys. Lett. A*, 223, 1996.

Marwan, N., Wessel, N., Meyerfeldt, U., Schirdewan, A., Kurths, J.: Recurrence Plot Based Measures of Complexity and its Application to Heart Rate Variability Data, *Phys. Rev. E*, 66(2), 2002.

Romano, M., Thiel, M., Kurths, J., von Bloh, W.: Multivariate Recurrence Plots, *Phys. Lett. A*, 330, 2004.

**Purpose** Plots the maximal correlation function.

**Syntax** `mcf(x,y [,w,t])`  
`mcor=mcf(x,y [,w,t])`  
`[time, mcor]=mcf(x,y [,w,t])`

**Description** Without any output arguments, `mcf` plots the maximal correlation function up to the maximal lag of `t` and by using a boxcar window size of  $2w + 1$ . Else, the maximal correlation function is stored in the vector `mcor` and its time scale in the vector `time`. If `w=[]`, the default boxcar window size is 11.

**Examples** `x=sin(0:.05:10)+.5*randn(1,201);`  
`y=cos(0:.05:10);`  
`mcf(x,y,[],20)`

**See Also** `ace`

**References** Breiman, L., Friedman, J. H.: Estimating Optimal Transformations for Multiple regression and Correltaion, J. Am. Stat. Assoc., Vol. 80, No. 391, 1985.

Voss, H., Kurths, J.: Reconstruction of nonlinear time delay models from data by the use of optimal transformations, Phys. Lett. A, 234, 1997.

**Purpose** GUI for data analysis programmes.

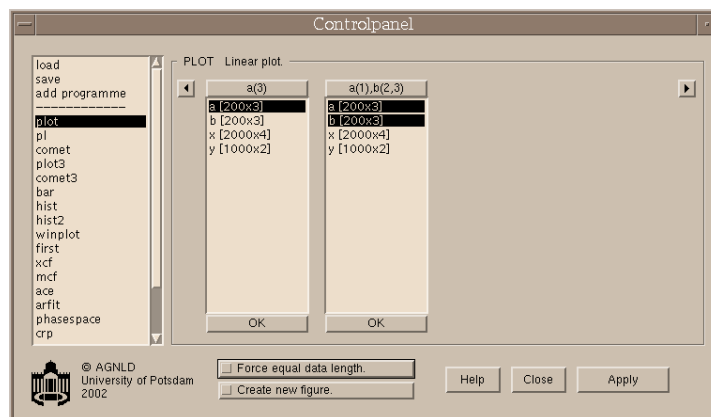
**Syntax** `mgui`

**Description** `mgui` starts a GUI and supplies Matlab programmes for their easy application to data which are in the Matlab workspace.

The presented programmes are stored in the file `mgui.rc` where own programmes can be added. Just include a line with the name of the programme and the minimal and maximal number of arguments, divided by a blank space or tabulator as a separator, e.g.

```
plot 1 4
```

If the embedded programmes provide an output, then this output will be stored in the variable `ans` in the Matlab workspace.



**Purpose** Histogram based mutual information.

**Syntax** `i=mi(x)`  
`i=mi(x1,...,xN)`  
`i=mi(x,...,xN,l)`  
`i=mi(x,...,xN,k,l)`  
`[i s]=mi(...)`  
`mi(...)`  
`mi(...,'param')`

**Description** Computes the mutual information between the vectors  $x_1, \dots, x_N$ . The auto mutual information can be computed by using only one vector. The arguments can be multi-column vectors. The result `i` will be a  $N \times N$  matrix.

`[i s]=mi(...)` computes the mutual information and the standard error (only for one and two arguments).

`mi(...)` without any output arguments opens a GUI for interactively changing the parameters.

By using the GUI, the mutual information can be stored into the workspace. If their standard error is available, they will be appended to the mutual information matrix as the last two columns (the stored matrix will have the size  $2 \times 4$ ).

`mi` without any arguments calls a demo (the same as the example below).

**Parameters** The parameters numbers of bins `k` and maximal lag `l` are optional. If the number of bins is not set, an amount of 10 will be used.

Additional parameters according to the GUI.

`'gui'` – Creates the GUI.  
`'nogui'` – Suppresses the GUI.  
`'silent'` – Suppresses all output.

Parameters not needed to be specified.

**Examples** `x=sin(0:.2:8*pi)'+.1*randn(126,1);`  
`mi(x,10,40)`

**Remark** Please note that the mutual information derived with `mi` slightly differs from the results derived with `migram`. The reason is that `mi` also considers estimation errors.

**See Also** `hist2`, `histn`, `entropy`

**References** Roulston, M. S.: Estimating the errors on measured entropy and mutual information, *Physica D*, 125, 1999.

**Purpose** Calculate windowed mutual information between two signals.

**Syntax** `i = migram(a,b,maxlag>window,noverlap,nbins)`  
`[i,l,t] = migram(...)`  
`i = migram(a,b)`  
`migram(a,b)`

**Description** `i = migram(a,b,maxlag>window,noverlap)` calculates the windowed mutual information between the signals in vector `a` and vector `b`. `migram` splits the signals into overlapping segments and forms the columns of `i` with their mutual information values up to maximum lag specified by scalar `maxlag`. Each column of `i` contains the mutual information function between the short-term, time-localized signals `a` and `b`. Time increases linearly across the columns of `i`, from left to right. Lag increases linearly down the rows, starting at `maxlag`. If lengths of `a` and `b` differ, the shorter signal is filled with zeros. If `n` is the length of the signals, `i` is a matrix with  $2*maxlag+1$  rows and

$k = \text{fix}((n-noverlap)/(window-noverlap))$

columns.

`i = migram(a,b,maxlag>window,noverlap,nbins)` calculates the mutual information based on histograms with the number of bins `nbins`.

`i = migram(...,'norm')` calculates the renormalised mutual information, which is  $i / \log(n_{bins})$  and ensures a value range  $[0 \dots 1]$ .

`[i,l,t] = migram(...)` returns a column of lag `l` and one of time `t` at which the mutual information is computed. `l` has length equal to the number of rows of `i`, `t` has length `k`.

`i = migram(a,b)` calculates windowed mutual information using default settings; the defaults are `maxlag = floor(0.1*n)`, `window = floor(0.1*n)`, `noverlap = 0` and `nbins = 10`. You can tell `migram` to use the default for any parameter by leaving it off or using `[]` for that parameter, e.g. `migram(a,b,[],1000)`.

`migram(a,b)` with no output arguments plots the mutual information using the current figure.

**Example** `x = cos(0:.01:10*pi)';`  
`y = sin(0:.01:10*pi)' + .5 * randn(length(x),1);`  
`migram(x,y)`

**Remark** Please note that the mutual information derived with `mi` slightly differs from the results derived with `migram`. The reason is that `mi` also considers estimation errors.

**See Also** `mi`, `corrgram`

## normalize

---

**Purpose** Normalizes data.

**Syntax** `y=normalize(x)`

**Description** Normalizes the matrix `x` to zero-mean and standard deviation of one ( $y = (x - \langle x \rangle) / \sigma_x$ ).

**Examples** `x=randn(100,1);`  
`std(x), mean(x)`  
`std(normalize(x)), mean(normalize(x))`

# phasespace

---

**Purpose** Computes phase space size.

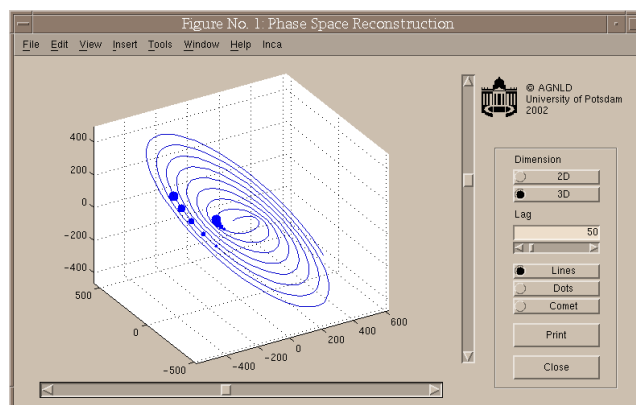
**Syntax** `phasespace(x)`  
`phasespace(x,y)`  
`phasespace(x,y,z)`

**Description** Shows the 3D phase space trajectory of the system which is presented by the observation `x`. The phase vectors are a reconstruction by using the time delay method (Takens, 1981). A GUI provides to change the embedding dimension to 2D.

`phasespace(x,y)` or `phasespace(x,y,z)` uses the one-column vectors `x`, `y` (and `z`) as the components of the phase space trajectory. The representation is 2D (3D) only and cannot be switched to the other representation.

`phasespace` without any arguments calls a demo (the same as the example below).

**Example** `phasespace(cos(0:.1:32).*[321:-1:1])`



**See Also** `fnn`, `pss`

**References** Takens, F.: Detecting Strange Attractors in Turbulence, Lecture Notes in Mathematics, 898, Springer, Berlin, 1981

**Purpose** Computes phase space size.

**Syntax** `pss(x)`  
`pss(x,m,t)`  
`[y z]=pss(...)`  
`[y z]=pss(...,'param')`

**Description** `pss(...)` computes the maximal phase space diameter of embedded data series `x` with the embedding parameters dimension `m` and lag `t`. A norm can be specified with an additional parameter.

`[y z]=pss(...)` computes the maximal `y` and the averaged `z` phase space diameter of embedded data series `x`.

**Parameters** Parameter for used norm.  
    'maxnorm'       – Maximum norm.  
    'euclidean'    – Euclidean norm (default).  
    'minnorm'       – Minimum norm.

**See Also** `phasespace`, `crp`, `crp2`

## trackplot

---

**Purpose** Estimates the line of synchronization of a cross recurrence plot.

**Syntax** `trackplot(x)`  
`trackplot(x,dx,dy)`  
`trackplot(x,dx,dy,'param')`  
`a=trackplot(...)`  
`[a b]=trackplot(...)`

**Description** `trackplot(x)` estimates the line of synchronization (LOS) in a cross recurrence plot `x`. The resulted path is exported to the workspace variable `t_out`. This command allows the interactive changing of estimation parameters.

`[a b]=trackplot(...)` estimates the LOS and stores it in `a`. The number of recurrence points met by the LOS is stored in `b(1)` and the number of lacks in the LOS is stored in `b(2)`.

**Parameters** The search of the LOS can be forced with the parameters `dx` and `dy`. An additional flag `param` allows to suppress the GUI (useful in order to use this programme by other programmes).

Suppressing the GUI.

`'gui'` – Creates the GUI and the output plot.  
`'nogui'` – Suppresses the GUI and the output plot.  
`'silent'` – Suppresses all output.

**Examples** `y=sin([1:900]*2*pi/67)';`  
`y2=sin(.01*([1:900]*2*pi/67).^2)';`  
`x=crp_big(y,y2,3,12,.1,'fan','nogui');`  
`trackplot(x,2,2)`

**See Also** `crp2`, `crp` and `crp_big`

**References** Marwan, N., Thiel, M., Nowaczyk, N.: Cross Recurrence Plot Based Synchronization of Time Series, *Nonlin. Proc. Geophys.* 9, 2002.

## trafo

---

**Purpose** Transforms data to a desired distribution.

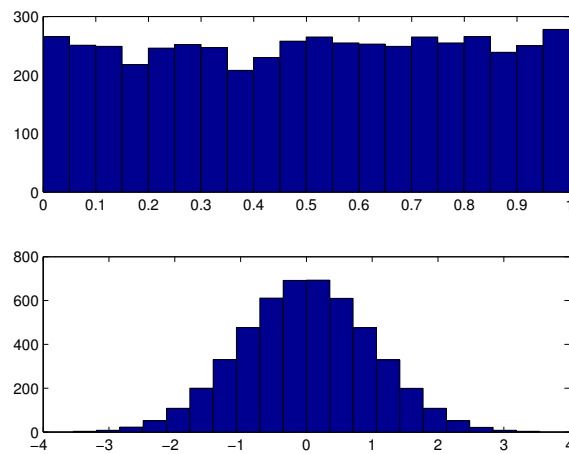
**Syntax** `y=trafo(x,a)`

**Description** `y=trafo(x,a)` transforms the data in vector `x` to data `y` of a desired distribution, where

`a=0` – normal distribution (default),  
`a=1` – uniform distribution,  
`a=2` – exponential distribution.

**Example**

```
x=rand(5000,1);  
subplot(2,1,1), hist(x,20)    % uniformly distributed  
y=trafo(x,0);  
subplot(2,1,2), hist(y,20)    % normally distributed
```



**Purpose** Mean trapping time and its distribution.

**Syntax** `a=tt(x)`  
`[a b]=tt(x)`

**Description** `a=tt(x)` computes the mean of the length of the vertical line structures in a recurrence plot, so called trapping time `tt`.

`[a b]=tt(x)` computes the `tt` and the lengths of the found vertical line structures, stored in `b`. In order to get the histogramme of the line lengths, simply call `hist(b,[1 max(b)])`.

**See Also** `crqa`, `crqaplot`, `dl`

# winplot

---

**Purpose** Windowed plot.

**Syntax** `winplot(x)`  
`winplot(x,w,ws)`  
`winplot(x,w,ws,flag)`  
`y=winplot(x,'parm')`

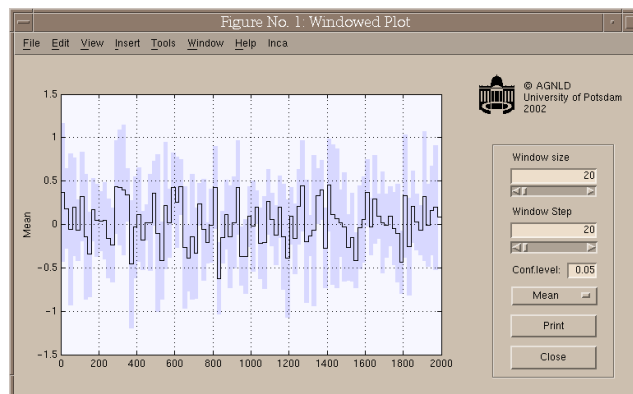
**Description** `winplot(x [,w,ws])` plots means or variances of the sub-vectors of vector `x`, which have the length `w` and are shifted by the step `ws`. `x` can be a two-column vector, where the first column would be the time-scale.

`winplot` without any arguments calls a demo (the same as the example below).

**Parameters** The optional parameter `flag` can determine the kind of the result, where `flag` can be either a string or a scalar:

- 'mean' or 1 – Mean (1st moment).
- 'var' or 2 – Variance (2nd moment).
- 'std' or 3 – Standard deviation.
- 'median' or 4 – Median.
- 'sqm' or 5 – Squared Mean.
- 'geo' or 6 – Geometric Mean.
- '3rd' or 7 – 3rd moment.
- 'skw' or 8 – Skewness.
- 'kur' or 9 – Kurtosis.

**Example** `winplot(randn(2000,1),20,20)`



**See Also** `plot`

## Plugin

---

**Description** A precompiled plugin for the computation of (cross/joint) recurrence plots and their quantification can be used for really long data series (several 10 000 data points). It may accelerate the computation as well.

**Usage** Download the corresponding installation script `plugininstall.m` for your system and put it any folder, where Matlab can find it. Call `plugininstall` from the Matlab commandline. You may check if it works with the command `rp_plugin`.

After installation, this plugin is used by the commands `crp`, `crp_big`, `crp2`, `jrp`, `crqa` and `jrqa`, if *Maximum norm*, *Euclidean norm*, *Minimum norm* or *Distance matrix* is used as a neighbourhood criterion. If two data vectors are used (for cross or joint recurrence plots), the plugin will only be used if both data vectors have the same length.

**Supported Systems** Currently the following systems are supported:

- True64 OSF1(5.1) on alpha
- HP-UX 11 on HP U9000
- Solaris 5.9 on Sun
- Linux on i686
- Linux on AMD Opteron 64
- Linux on Intel Itanium 2
- Dos/Win on x86

## Error Handling

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**Error Support** If an error occurs, an extensive error report will be supplied in the file `error.log`. Please send us this error report and provide a brief description of what you were doing when this problem occurred. E-mail or FAX this information to us at:

E-mail: [marwan@agnld.uni-potsdam.de](mailto:marwan@agnld.uni-potsdam.de)

Fax: ++49 +331 977 1142

Thank you for your assistance.

**Error Codes** The following error codes mark the location of the error in the programmes.

code	location in programme
0	ok
1	initialization
2	create crp figure
3	create control gui
4	vectorswitch/ vectorexclude
5	fit dimension display
6	unthresh
7	stretch
81	change colormap
82	change colormap scale
9	store
91	print
101	close all
102	smart close
11	init computation
111	local CRP, fixed distance maximum norm
112	local CRP, fixed distance euclidean norm
113	local CRP, fixed distance minimum norm
114	local CRP, normalized distance euclidean norm
115	local CRP, fixed neighbours amount
116	local CRP, interdependent neighbours method
117	order matrix
117	global CRP
12	show local CRP
13	show global CRP
14	set handles and axes ratios
15	LOS store
16	LOS move
161	LOS move end
17	LOS clear
18	LOS set
19	LOS search
191	looks for the beginning of the diagonal LOS
192	start estimation of the LOS
193	looks for the existence of the next recurrence point
194	determines the coordinates of the next recurrence point
195	determines the local width of the diagonal LOS
196	compute the mean of the diagonal LOS
197	DTW algorithm, seek process

### *continuation*

198	DTW algorithm, fixed points
199	show LOS
20	CRQA computation
30	CRQA plot
90	installation