



Theoretical concept for initialization of forest stands

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1. Initialization using averaged stand data

1.1. Data requirement

The initialization requires per stand

- *species type*
- *averaged diameter at breast height D_g*
- *averaged height H_g*
- *age and*
- *basal area G of the stand.*

Species specific parameters of various functions are necessary:

- $p_0 - p_4$ for Weibull distribution function, equation (1.2)
- c_1, c_2 for bole height function, equation (1.9)
- $a_0 - a_2$ for height function, equation (1.4)
- $b_0 - b_2$ for height function, equation (1.4)
- $kc_0 - kc_2$ for height function, equation (1.4)
- wk_1, wk_2 for height function, equation (1.6)

1.2. Method

1.2.1. Generating of single tree data

A capping limit parameter $L = 7$ is defined. If $(D_g - L) < 3$ then $L = D_g - 4$. A maximum diameter d_{max} is estimated from D_g by a function of (Gerold, 1990):

$$D_{max} = 8.2 + 1.8D_g - 0.1D_g^2 \quad (1.1)$$

Two parameters b and c of a Weibull distribution are calculated:

$$\begin{aligned} b &= p_0 + p_1 \cdot D_g \\ c &= p_2 + p_3 \cdot D_g + p_4 \cdot D_{max} \end{aligned} \quad (1.2)$$

The generation of a single tree with the diameter d_{bh} is realized by a number x equally distributed between 0 and 1 and the following function (Weibull distribution):

$$d_{bh} = b \cdot \left(\left(\frac{L}{b} \right)^c - \log(1-x) \right)^{\frac{1}{c}} \quad (1.3)$$

The height of the tree with diameter d_{bh} is estimated by different height functions.

For pine and birch the height function the height function from Kuleschis (see (Gerold, 1990)) is used with the parameters ka, kb, kc :



$$\begin{aligned}
 ka &= 1 - (a_0 + a_1 D_g + a_2 D_g^2) \\
 kb &= 1 - (b_0 + b_1 D_g + b_2 D_g^2) \\
 kc &= 1 - (kc_0 + kc_1 D_g + kc_2 D_g^2)
 \end{aligned}
 \tag{1.4}$$

and the function:

$$H = H_g \cdot \left(ka + \frac{kb}{\left(d_{bh} + \frac{D_g}{2} \right)} D_g + \frac{kc}{\left(d_{bh} + \frac{D_g}{2} \right)^2} D_g^2 \right)
 \tag{1.5}$$

For other species (beech, spruce, oak) the height function of (Weimann, 1980) is applied with the parameter wf:

$$wf = wk_1 + wk_2 \cdot H_g
 \tag{1.6}$$

For the case d_{bh} is greater or equal $D_g - H_g/2$ the following function is used:

$$H = H_g + wf \cdot (\log(H_g - D_g + d_{bh}) - \log(H_g))
 \tag{1.7}$$

Otherwise:

$$= \left(H_g + wf \cdot \left(\log\left(\frac{H_g}{2}\right) - \log(H_g) \right) - 1.3 \right) \left(\frac{d_{bh}}{\left(D_g - \frac{H_g}{2} \right)} \right)^{0.5} + 1.3
 \tag{1.8}$$

For Douglas fir (Nagel, 2002) and Eucalyptus (Medhurst et al., 1999; Ranatunga et al., 2008) specific height functions are implemented .

The bole height H_b of the tree is estimated according to a function of (Nagel, 1995):

$$H_b = H \cdot \left(1 - e^{\left(-1 \left(c_1 + c_2 \frac{H}{d_{bh}} \right)^2 \right)} \right)
 \tag{1.9}$$

Trees are generated by this method until the basal area G is reached. For Eucalyptus the bole height is estimated by a function of (Nutto et al., 2006):



$$H_b = -5.12 - 0.407 \cdot d_{bh} + 1.193 \cdot H \quad (1.10)$$

1.2.2. Building cohorts

The generated trees are classified into tree cohorts. The maximum number of classes is 60. The class width is calculated from the maximum and minimum DBH.

The further initial data of the cohorts are estimated as follows.

a) *Parameter rsap*

A parameter rsap is estimated, rsap describes the fraction of wood which is sapwood.

b) *The height of sapwood pipes H_s and stem wood*

H_s is calculated as

$$H_s = \frac{2H_{bc}}{3} + \frac{H}{3} \quad (1.11)$$

c) *The stem volume VD*

VD is calculated with different function per species.

For Douglas fir (Nagel, 2002) the following functions are used:

$$HFD = \frac{-200.31914}{H \cdot d_{bh}^2} + \frac{0.8734}{d_{bh}} - 0.0052 \cdot \log(d_{bh}^2) + \frac{7.3594}{H \cdot d_{bh}} + 0.46155 \quad (1.12)$$

and for other species (SILVA, (Pretzsch et al., 2002)):

$$\begin{aligned} k1 &= s1 + s2 \cdot \log(DBH) + s3 \cdot \log(DBH)^2 \\ k2 &= s4 + s5 \cdot \log(DBH) + s6 \cdot \log(DBH)^2 \\ k3 &= s7 + s8 \cdot \log(DBH) + s9 \cdot \log(DBH)^2 \end{aligned} \quad (1.13)$$

with s1-s9 species-specific parameters

$$HFD = e^{k1+k2 \cdot \log(H)+k3 \cdot \log(H)^2} \quad (1.14)$$

The resulting stem volume is:

$$VD = \frac{HFD \cdot \pi \cdot d_{bh}^2}{4000} \quad (1.15)$$

For pine stem volume is calculated as follows:



$$VD = e^{(p_1 + p_2 \log(d_{bh}) + p_3 \log(H))} \quad (1.16)$$

For Eucalyptus a function of (Binkley et al., 2002) is used:

$$VD = 0.00005447 \cdot d_{bh}^{1.921157} \cdot \left(\frac{H}{100}\right)^{0.950581} \quad (1.17)$$

or a function of (Stape et al., 2010) depending on the site is used:

$$VD = \frac{p_1 d_{bh}^{p_2} \cdot \left(\frac{H}{100}\right)^{p_3}}{500} \quad (1.18)$$

p_1, p_2, p_3 – site depending parameters

d) *Sapwood area* A_s

If H_b less than 137 the diameter at base of the crown d_{cb} is calculated:

$$d_{cb} = \frac{d_{bh}}{H} (137 - H_b) + d_{bh} \quad (1.19)$$

and the sapwood area A_s

$$A_s = \frac{\pi}{4} \cdot d_{cb}^2 \cdot rsap \quad (1.20)$$

If H_b is greater than 137 cm

$$A_s = \frac{\pi}{4} d_{bh}^2 \cdot rsap \quad (1.21)$$

$$rsap = \frac{A_s}{\frac{\pi}{4} \cdot d_{bh}^2}$$

e) *Sapwood biomass* M_s

$$M_s = \rho_s \cdot A_s \cdot H_s \quad (1.22)$$

ρ_s - sapwood density

f) *cross sectional area of heart wood at crown base* A_{hc} *and cross sectional area of heart wood at stem base* A_{hb}

If H_b is less than 137 cm then



$$A_{hc} = \frac{\pi}{4} d_{cb}^2 - A_s \quad (1.23)$$

$$A_{hb} = \frac{\pi}{4} \left(\frac{d_{cb} \cdot a}{\tau_{max}} \right)^2 - A_s \quad (1.24)$$

τ_{max} – parameter

a - age

If H_b is greater than 137 cm then

$$A_{hc} = \frac{\pi}{4} \cdot d_{bh}^2 \cdot (1 - rsap) \cdot 0.04 \quad (1.25)$$

as initial value for the calculation of A_{hc} with a Newton algorithm. This algorithm uses a function F (in SR fdfahc):

$$F = M_{bio} - \rho_s \cdot VD \quad (1.26)$$

VD – estimation of stem mass by yield table functions

M_{bio} –total stem biomass

Stem biomass is considered as truncated cone with base area A_{bs} and the diameter d_{cb} and the height H_b and a cone with height $H-H_b$ and ground base $A_s + A_{hc}$ (A_{bs}). The ground base of heartwood A_{hb} is:

$$A_{hb} = A_{bs} - A_s \quad (1.27)$$

The heartwood area at crown base $A_{hc}(x)$ has to calculate.

The biomass of stem is calculated as sum of sapwood and heartwood biomass, which is calculated from the truncated cone with the ground base area A_{hb} , the upper area A_{hc} and the height H_b and the cone with the height $H-H_b$ and the base area A_{hc} .

Sapwood biomass is calculated:

$$M_s = A_s \frac{1}{3} (2H_b + H) \quad (1.28)$$

Heartwood biomass:

$$M_{hw} = \frac{1}{3} x (H - H_b) + \frac{1}{3} H_b \left(x + \left(\frac{\pi}{4} d_{bs}^2 - A_s + \sqrt{x \left(\frac{\pi}{4} d_{bs}^2 - A_s \right)} \right) \right) \quad (1.29)$$

The total stem biomass is:



$$\begin{aligned}
 M_{bio} &= M_s + M_{hw} \\
 &= A_s \frac{1}{3} (2H_b + H) + \frac{1}{3} x (H - H_b) + \frac{1}{3} H_b \left(x + \left(\frac{\pi}{4} d_{bs}^2 - A_s + \sqrt{x \left(\frac{\pi}{4} d_{bs}^2 - A_s \right)} \right) \right) \\
 &= \frac{1}{3} \left(H(A_s + x) + H_b \left(2A_s - x + x - A_s + \frac{\pi}{4} d_{bs}^2 + \sqrt{x \left(\frac{\pi}{4} d_{bs}^2 - A_s \right)} \right) \right) \\
 &= \frac{1}{3} \left(H(A_s + x) + H_b \left(A_s + \frac{\pi}{4} d_{bs}^2 + \sqrt{x \left(\frac{\pi}{4} d_{bs}^2 - A_s \right)} \right) \right)
 \end{aligned} \tag{1.30}$$

The diameter d_{bs} of the area A_{bs} is calculated as follows.

The diameter of the upper base area of the truncated cone with height H_b of the total biomass is d_{cb} :

$$d_{cb} = \sqrt{\frac{4}{\pi} (A_s + x)} \tag{1.31}$$

According to the Strahlensatz the relations between d_{bs} und d_{bh} are:

$$\frac{D_{bh} - d_{cb}}{d_{bs} - d_{cb}} = \frac{H_b - H_d}{H_b} \tag{1.32}$$

H_d - Höhe des DBH (137 cm)

$$\begin{aligned}
 \frac{d_{bh} - d_{cb}}{d_{bs} - d_{cb}} &= \left(1 - \frac{H_d}{H_b} \right) \\
 d_{bs} - d_{cb} &= \frac{d_{bh} - d_{cb}}{\left(1 - \frac{H_d}{H_b} \right)} \\
 d_{bs} &= \frac{d_{bh} - d_{cb}}{\left(1 - \frac{H_d}{H_b} \right)} + d_{cb} \\
 &= \frac{d_{bh} - d_{cb} + d_{cb} \left(1 - \frac{H_d}{H_b} \right)}{\left(1 - \frac{H_d}{H_b} \right)} = \frac{d_{bh} - d_{cb} \frac{H_d}{H_b}}{\left(1 - \frac{H_d}{H_b} \right)}
 \end{aligned} \tag{1.33}$$

Together with equation (1.32) it yields:



$$d_{bs} = \frac{d_{bh} - \sqrt{\frac{4}{\pi} (A_s + x) \frac{H_d}{H_b}}}{\left(1 - \frac{H_d}{H_b}\right)} \quad (1.34)$$

and

$$\frac{dd_{bs}}{dx} = \frac{-H_d}{H_b \pi \left(1 - \frac{H_d}{H_b}\right) \sqrt{\frac{1}{\pi} (A_s + x)}} \quad (1.35)$$

A solution for x is found by assuming F(x)=0 with the Newton algorithm.

g) *Stem biomass*

Calculated as heartwood cone above H_b and a truncated cone below H_b (first term) plus a coat of sapwood (second term):

$$\begin{aligned} M_{bio} &= \rho_s \frac{1}{3} \left(A_{hc} H + H_b \left(A_{hb} + \sqrt{A_{hb} A_{hc}} \right) \right) + \rho_s A_{sap} \left(\frac{2}{3} H + \frac{1}{3} H_b \right) \\ &= \rho_s \left(\frac{1}{3} H \cdot (A_s + A_{hc}) + \frac{1}{3} H_b \cdot \left(2A_s + A_{hb} + \sqrt{A_{hb} A_{hc}} \right) \right) \\ &= \rho_s \cdot \frac{1}{3} \left(H \cdot (A_s + A_{hc}) + H_b \cdot \left(2A_s + A_{hc} + \sqrt{A_{hb} A_{hc}} \right) \right) \end{aligned} \quad (1.36)$$

h) *Heart wood biomass M_{hw}*

$$M_{hw} = M_{bio} - M_s \quad (1.37)$$

i) *Foliage biomass M_f and foliage area A_{fol}*

$$M_f = \eta_s \cdot A_s \quad (1.38)$$

$$A_{fol} = M_f \cdot s_{min} + 0.5 \cdot s_{a,c} \quad (1.39)$$

η_s – species-specific foliage to sapwood area relationship

s_{min} – minimum specific one-side leaf area

$s_{a,c}$ – light depended specific one-side leaf area

j) *Fine root biomass M_r*



$$M_r = M_f \quad (1.40)$$

as rough estimate.

2. Initialization using single tree data

2.1. Data requirements

The initialization requires per tree the following data:

- *Patch size*
- *species type*
- d_{bh} (mm)
- H (m)
- H_b (mandatory)
- *age*

2.2. Method

The given data are classified into diameter classes to build cohorts. The class width is 1cm, but could be changed. If H_b is not available, the functions (1.9) and (1.10) are used to calculate H_b . For each diameter class an averaged d_{bh} , H and H_b and the number of trees are calculated as values of the cohort. Further on, all cohort variables, described in chapter 1.2.2, are calculated.

3. Initialization of saplings

3.1. Data requirements

If the height of planted trees are less than a specific limit (e.g. 200 cm) the DBH is not available. In this case a more simple way of initialization of cohorts is realised. It requires:

- *Species type*
- *Age*
- *Number of trees N_p*
- *Mean height H*
- *Minimum height H_{min}*
- *Standard deviation of height σ_H*

3.2. Method

3.2.1. Height and number of trees per cohort

A number of saplings is generated, standard values of plants per hectare are given in the model, as well as the other required data. A number of cohorts N_{class} is calculated in different way, e.g. the integer of H is used or a number is fixed ($N_{class} = 20$). For each sapling class/ cohort c_i the height H_i is calculated:



$$H_i = H_{min} + (i - 1) \quad 3-1$$

The number of trees per cohort I NH_i is calculated:

$$NH_i = \frac{1}{\sqrt{2 \cdot \pi} \cdot \sigma_H} \cdot e^{-\frac{(H_i - H)^2}{2 \cdot \sigma_H^2}} \quad 3-2$$

The the total number N_{tot} is

$$N_{tot} = \sum_{i=1}^{N_{class}} NH_i \quad 3-3$$

And following the number of trees per class/ cohort N_i is recalculated:

$$N_i = NH_i \cdot \frac{N_p}{N_{tot}} \quad 3-4$$

3.2.2. Further initialization of cohorts

For the calculation of sapwood biomass X_{sap,i} per sapling cohort I the roots of the following equation has to solve

$$XH_{sap,i} = p_3 \cdot x^2 + p_2 \cdot x + p_1 - \log(H_i) \quad 3-5$$

$$X_{sap,i} = \frac{10^{XH_{sap,i}}}{100000} \quad 3-6$$

p₁, p₂, p₃ – species specific parameters

The foliage biomass X_{fol,i} is calculated as follows:

$$X_{fol,i} = p_a \cdot X_{sap,i}^{p_b} \quad 3-7$$

p_a, p_b – species-specific parameters

and the fine root biomass X_{frt,i} is estimated:

$$X_{frt,i} = X_{fol,i} \quad 3-8$$

The parametrization of saplings of the main tree species in 4C is based on data found in (Barigah et al., 1994; Bond-Lamberty et al., 2002; Dohrenbusch, 1997; Hauskeller-Bullerjahn, 1997; Mailly and Kimmins, 1997; Ter-Mikaelian and Korzukhin, 1997; Van Hees, 1997).

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